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Twistor string as tensionless superstring¹

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Abstract

We give a brief review of the twistor string approach to supersymmetric Yang-Mills theories with an emphasis on the different formulations of (super)string models in supertwistor space and their superspace form. We discuss the classical equivalence among the Siegel closed twistor string and the Lorentz harmonics formulation of the ($N = 4$) tensionless superstring, and notice the possible relation of the twistor string to the $D = 10$ Green-Schwarz superstring action, as well as to models in the enlarged, tensorial superspaces that are relevant in higher spin theories.

1 MHV Yang-Mills amplitudes, twistors and super-twistors

At the end of 2003, E. Witten looked again [1] at the connection between $N=4$ supersymmetric Yang-Mills (SYM) theory and string theory using the twistor approach [2]. In contrast to the conventional AdS/CFT correspondence [4], the above connection establishes a link between the weak coupling limits on both sides and, hence, it can be checked perturbatively. This led to the development of a new technique to compute gauge theory amplitudes [3] (the Cachazo-Svrček-Witten [CSW] or MHV diagram rules, see below) and renewed the interest in the Penrose twistor program [2] of replacing spacetime by twistors.

1.1. MHV amplitudes through bosonic spinors

The amplitude for the scattering of n gauge bosons with 2 positive and $(n-2)$ negative helicities reads

$$A(1, 2, \dots, n) = 2^{n/2} i g^{n-2} \text{Tr}(t^{a_1} \dots t^{a_n}) \frac{\langle IJ \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle} \quad (1)$$

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(see [5, 6] and refs. therein), where g is the YM coupling constant, $Tr(t^{a_1}, \dots, t^{a_n})$ is the trace of the product of n gauge group generators, and the one-particle matrix elements $\langle 12 \rangle, \dots, \langle n1 \rangle$ are expressed through the contraction of pairs of bosonic spinors, a spinor for each gluon state, *e.g.*,

$$\langle 12 \rangle = \lambda^{\alpha 1} \lambda_{\alpha}^2 = \epsilon^{\alpha\beta} \lambda_{\beta}^1 \lambda_{\alpha}^2 \equiv -\lambda^{\alpha 2} \lambda_{\alpha}^1 = -\langle 21 \rangle . \quad (2)$$

Specifically, I and J in (1) ($A(1, 2, \dots, n) \propto \langle IJ \rangle^4$) refer to two positive helicity gluons, while the gluons with $n \neq I, J$ are assumed to have negative helicity. Notice that the maximally helicity violating or MHV amplitude (1) is holomorphic: it depends on the λ 's through their contractions $\langle 12 \rangle$, etc. In contrast, the complex conjugate spinors $\bar{\lambda}_{\alpha}^1 := (\lambda_{\alpha}^1)^*$, \dots , $\bar{\lambda}_{\alpha}^n := (\lambda_{\alpha}^n)^*$, *etc.*, the contractions of which are denoted by $[1, 2] := \bar{\lambda}^{1\dot{\alpha}} \bar{\lambda}_{\dot{\alpha}}^2 \equiv -[2, 1]$, are not present in (1).

One may ask, why the n particle amplitudes can be expressed in terms of just n bosonic spinors λ_{α}^i ? The answer is that a bosonic spinor can be used to describe the *on-shell* momentum and the helicity that characterize (modulo internal quantum numbers) the on-shell state of a massless particle. The momentum of such particle is lightlike, $p^2 := p_{\mu} p^{\mu} = 0$, a condition that is solved in spinor space by [2]

$$p_{\alpha\dot{\alpha}} := p_{\mu} \sigma_{\alpha\dot{\alpha}}^{\mu} = \lambda_{\alpha} \bar{\lambda}_{\dot{\alpha}} \quad \Leftrightarrow \quad p_{\mu} = \frac{1}{2} \lambda \sigma^{\mu} \bar{\lambda} := \frac{1}{2} \sigma_{\mu\alpha\dot{\alpha}} \lambda^{\alpha} \bar{\lambda}^{\dot{\alpha}} . \quad (3)$$

The light-likeness of the vector p_{μ} given by the Penrose formula (3) follows from $p^2 \delta_{\alpha}^{\beta} = p_{\alpha\dot{\alpha}} \tilde{p}^{\dot{\alpha}\beta} = \lambda_{\alpha} \bar{\lambda}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} \lambda^{\beta}$, which vanishes identically.

The polarization vectors $\varepsilon^{\mu\pm}$ of the positive and negative helicity particles are defined to be lightlike and orthogonal to the lightlike momentum, $(\varepsilon^{\pm})^2 = 0$, $\varepsilon^{\mu\pm} p_{\mu} = 0$. They are expressed by

$$\varepsilon_{\alpha\dot{\alpha}}^{-i} = \lambda_{\alpha}^i \bar{u}_{\dot{\alpha}} / [\bar{\lambda}^i \bar{u}] \quad , \quad \varepsilon_{\alpha\dot{\alpha}}^{+i} = u_{\alpha} \bar{\lambda}_{\dot{\alpha}}^i / \langle u, \lambda^i \rangle \quad , \quad (4)$$

using a constant reference spinor $\bar{u}_{\dot{\alpha}} (u_{\alpha} = (\bar{u}_{\dot{\alpha}})^*)$. The relativistic invariance condition corresponds to the requirement that the observable quantities are independent of the choice of u .

Thus, all the kinematical information on the *on-shell* state of a gauge boson, its helicity and its lightlike momentum, are encoded in a single bosonic spinor. This is the reason why the scattering amplitude for n gauge bosons can be condensed in an expression written in terms of bosonic spinors, as (1) above.

1.2. The Cachazo-Svrček-Witten or MHV diagram technique inspired by the twistor string

The CSW diagram technique [3] consists in cutting a Feynman diagram into MHV pieces (which is always possible [3]) and then treating them as vertices connected by scalar propagators. Clearly, there is an immediate problem: by cutting a Feynman diagram into MHV pieces, one gets generally subdiagrams in which one or more legs correspond to virtual particles, *i.e.* particles that are off-shell and for which the basic Penrose representation (3) does not hold. The prescription proposed in [3] and checked to be true inside and beyond [6] the domain of the original $N=4$ supersymmetric Yang-Mills context, is to associate to

a virtual particle the bosonic spinor defined by $\lambda_\alpha(p) = p_{\alpha\dot{\alpha}}\bar{\omega}^{\dot{\alpha}}$, where $\bar{\omega}^{\dot{\alpha}}$ is an arbitrary reference spinor. Relativistic invariance then requires that the amplitude A is independent of the choice of the reference spinor, $\partial A/\partial\bar{\omega} = 0$. This condition has been shown to hold for tree and one-loop diagrams in $N=4$ SYM theory, and checked for some two-loop and some non- and less supersymmetric theories [6].

The equivalence of the MHV and the Feynman diagram calculus was originally proved for the tree diagrams of $N=4$ SYM theory [3]. It was then extended to one-loop diagrams [3] and also checked for some higher-loop ones as well as for less supersymmetric ($N=2$, $N=1$ and non-supersymmetric $N=0$) YM theories (see [6] for a recent review). However, the original version was developed for $N=4$ SYM theories and was inspired by the twistor string model [1], which is a string model formulated in the space of $N=4$ supertwistors. The action principles which lie beyond the twistor string [1, 7, 8] and its spacetime (superspace) formulation will be our main subject here.

To begin, we briefly address two questions: 1) what is a twistor? and 2) what is a supertwistor?

1.3 Penrose twistors and Ferber supertwistors

A twistor [2] can be understood as a Dirac spinor. It has four complex components in two Weyl spinors, $\Upsilon^{\hat{\alpha}} = (\lambda_\alpha, \mu^{\dot{\alpha}}) \in \mathbb{C}^4$, and provides a spinorial representation for the conformal group $SO(2, 4)$ as well as a fundamental representation for the (locally isomorphic) $SU(2, 2) = Spin(2, 4)$ group.

Twistor space is usually considered as a complex projective space,

$$\Upsilon^{\hat{\alpha}} \sim z\Upsilon^{\hat{\alpha}} \quad \Rightarrow \quad \Upsilon^{\hat{\alpha}} = (\lambda_\alpha, \mu^{\dot{\alpha}}) \in \mathbb{CP}^3 \quad . \quad (5)$$

The reason for the identification $\Upsilon^{\hat{\alpha}} \sim z\Upsilon^{\hat{\alpha}}$ of twistors that differ by a complex factor $z \in \mathbb{C} \setminus \{0\}$ is the obvious complex scale invariance of the Penrose incidence relation

$$\mu^{\dot{\alpha}} = x^{\dot{\alpha}\alpha}\lambda_\alpha, \quad x^{\dot{\alpha}\alpha} := x^\mu \tilde{\sigma}_\mu^{\dot{\alpha}\alpha}, \quad (6)$$

which defines a spacetime point x^μ or, more precisely, a lightlike line in Minkowski space, $\hat{x}^{\dot{\alpha}\alpha}(\tau) = x^{\dot{\alpha}\alpha} + \tau \lambda^\alpha \bar{\lambda}^{\dot{\alpha}}$. Eq. (6) is the general solution for the ('helicity') constraint $\bar{\Upsilon}_{\dot{\alpha}} \Upsilon^{\dot{\alpha}} := \bar{\lambda}_{\dot{\alpha}} \mu^{\dot{\alpha}} - \bar{\mu}^\alpha \lambda_\alpha = 0$.

Ferber *supertwistors* [9], $\Upsilon^\Sigma := (\Upsilon^{\hat{\alpha}}, \eta_i) = (\lambda_\alpha, \mu^{\dot{\alpha}}, \eta_i) \in \mathbb{C}^{(4|N)}$, ($i = 1, \dots, N$), carry a fundamental representation of $SU(2, 2|N)$. They include N fermionic variables η_i , $i = 1, \dots, N$, in addition to the Penrose twistor $\Upsilon^{\hat{\alpha}}$. Under the $\Upsilon^\Sigma \sim z\Upsilon^\Sigma$ equivalence relation, supertwistors become homogeneous coordinates of the complex projective superspace $\mathbb{CP}^{(3|N)}$,

$$\Upsilon^\Sigma := (\Upsilon^{\hat{\alpha}}, \eta_i) = (\lambda_\alpha, \mu^{\dot{\alpha}}, \eta_i) \in \mathbb{CP}^{(3|N)}, \quad \eta_i \eta_j = -\eta_j \eta_i, \quad i = 1, \dots, N. \quad (7)$$

The scaling $\Upsilon^\Sigma \sim z\Upsilon^\Sigma$ appears now as a symmetry of the Penrose-Ferber incidence relations,

$$\mu^{\dot{\alpha}} = x_L^{\dot{\alpha}\alpha} \lambda_\alpha, \quad \eta_i = \theta_i^\alpha \lambda_\alpha \quad x_L^{\dot{\alpha}\alpha} := x_L^\mu \tilde{\sigma}_\mu^{\dot{\alpha}\alpha} := x^{\dot{\alpha}\alpha} + 2i\theta_i^\alpha \bar{\theta}^{\dot{\alpha}i}. \quad (8)$$

These involve the coordinates $Z^M := (x^\mu, \theta_i^\alpha, \bar{\theta}^{\dot{\alpha}i})$ of N -extended $D=4$ superspace and define a $(1|N)$ -dimensional subsuperspace $\mathbb{R}^{(1|N)}$,

$$\hat{x}^{\dot{\alpha}\alpha} = x^{\dot{\alpha}\alpha} + \tau \lambda^\alpha \bar{\lambda}^{\dot{\alpha}} + 2[i\kappa_i \bar{\theta}^{\dot{\alpha}i} \lambda^\alpha + c.c.] , \quad \hat{\theta}_i^\alpha = \theta_i^\alpha + \kappa_i \lambda^\alpha , \quad \{(\tau, \kappa^i)\} = \mathbb{R}^{(1|N)} , \quad (9)$$

where the κ_i are N fermionic parameters. This $\mathbb{R}^{(1|N)}$ is the Sorokin-Tkach-Volkov-Zheltukhin worldline superspace [10] (or *superworldline*, first introduced in the context of the spinning superparticle [13]), the simplest example of the superworldvolumes of the superembedding approach to superbranes [11, 12].

2 (Super)twistor string action(s)

The basic worldsheet fields of the twistor string models are the supertwistors (7). At present there are three main versions of the twistor string action (see [14] for further discussion and references): (i) the constrained $\mathbb{CP}^{(3|4)}$ sigma model by Witten [1]; (ii) the open string model by Berkovits [7] involving *two* supertwistors; and (iii) the simplest one, proposed by Siegel in [8], described by the closed string action

$$S = \int_{W^2} e^{++} \wedge \tilde{\Upsilon}_\Sigma \nabla \Upsilon^\Sigma + d^2 \xi L_G = \int d^2 \xi [\sqrt{|\gamma(\xi)|} \tilde{\Upsilon}_\Sigma(\xi) \nabla_{--} \Upsilon^\Sigma(\xi) + L_G] . \quad (10)$$

Here $\tilde{\Upsilon}_\Sigma := (\Upsilon^\Pi)^* \Omega_{\Pi\Sigma} = (\bar{\lambda}_{\dot{\alpha}}, -\bar{\mu}^\alpha; 2i\bar{\eta}^i)$ is the $SU(2, 2|N)$ -adjoint of Υ^Σ , $e^{\pm\pm} = d\xi^m e_m^{\pm\pm}(\xi)$ are the worldsheet zweibein one-forms and $e^{++} \wedge e^{--} = d^2 \xi \sqrt{|\gamma|}$ is the invariant surface element of the string worldsheet W^2 . The covariant derivative $\nabla = e^{++} \nabla_{++} + e^{--} \nabla_{--} = d - iB$ involves the $U(1)$ -connection B , which serves as a Lagrange multiplier for the constraint

$$\tilde{\Upsilon}_\Sigma \Upsilon^\Sigma = \bar{\lambda}_{\dot{\alpha}} \mu^{\dot{\alpha}} - \bar{\mu}^\alpha \lambda_\alpha + 2i\bar{\eta}^i \eta_i = 0 . \quad (11)$$

Finally, L_G in (10) is the lagrangian for the worldsheet fields used to construct the Yang-Mills symmetry current. As noted in [7], one can use *e.g.*, the worldsheet fermionic fields Ψ^I in the fundamental representation of the gauge group. Then, $d^2 \xi L_G = \frac{1}{2} e^{++} \wedge (\bar{\psi}_I d\psi^I - d\bar{\psi}_I \psi^I)$ in the notation of [14].

The lagrangian of the open string model (ii) by Berkovits [7] is given by $S = \int_{W^2} e^{++} \wedge \tilde{\Upsilon}_\Sigma^- \nabla(\Upsilon^{-\Sigma}) - e^{--} \wedge \tilde{\Upsilon}_\Sigma^+ \nabla(\Upsilon^{+\Sigma}) + \int_{W^2} d^2 \xi (L_G^L + L_G^R)$. It contains two supertwistors, one left-moving $\Upsilon^{-\Sigma}$ and one right-moving $\Upsilon^{+\Sigma}$, and also two copies of the ‘YM current’ degrees of freedom, which are ‘glued’ by boundary conditions on ∂W^2 . The lagrangian form integrated over the open worldsheet W^2 is actually the sum of Siegel’s lagrangian in (10) and its right-moving counterpart. Finally, the original action for the $\mathbb{CP}^{(3|4)}$ twistor string model (i) by Witten [1], expressed in the present notation, can be found in [14].

3 The twistor string as a tensionless superstring

As it was shown in [14], an equivalent form of the action (10) is given by the tensionless superstring action from [15, 16] (called twistor-like Lorentz-harmonics formulation of the

null superstring for reasons explained in [14]). This means that, ignoring its YM part, the action (10) can be written in $D = 4$, $N = 4$ superspace as

$$S = \int_{W^2} e^{++} \wedge \hat{\Pi}^{\dot{\alpha}\alpha} \bar{\lambda}_{\dot{\alpha}} \lambda_{\alpha} = \int_{W^2} e^{++} \wedge (dx^{\dot{\alpha}\alpha} - id\theta_i^{\alpha} \bar{\theta}^{\dot{\alpha}i} + i\theta_i^{\alpha} d\bar{\theta}^{\dot{\alpha}i}) \bar{\lambda}_{\dot{\alpha}} \lambda_{\alpha}, \quad (12)$$

where $\hat{\Pi}^{\dot{\alpha}\alpha} = d\tau \Pi_{\tau}^{\dot{\alpha}\alpha} + d\sigma \Pi_{\sigma}^{\dot{\alpha}\alpha}$ is the pull-back to the worldsheet W^2 of the flat supervielbein on $D = 4$, $N = 4$ superspace, $\Pi^{\dot{\alpha}\alpha} := dx^{\dot{\alpha}\alpha} - id\theta_i^{\alpha} \bar{\theta}^{\dot{\alpha}i} + i\theta_i^{\alpha} d\bar{\theta}^{\dot{\alpha}i}$. This action possesses an irreducible κ -symmetry

$$\begin{aligned} \delta_{\kappa} x^{\dot{\alpha}\alpha} &= i\delta_{\kappa} \theta_i^{\alpha} \bar{\theta}^{\dot{\alpha}i} - i\theta_i^{\alpha} \delta_{\kappa} \bar{\theta}^{\dot{\alpha}i}, & \delta_{\kappa} \theta_i^{\alpha} &= \kappa_i \lambda^{\alpha}, & \delta_{\kappa} \bar{\theta}^{\dot{\alpha}i} &= \bar{\kappa}^i \bar{\lambda}^{\dot{\alpha}}, \\ \delta_{\kappa} \lambda^{\alpha} &= \delta_{\kappa} \bar{\lambda}^{\dot{\alpha}} = \delta_{\kappa} e^{++} = 0. \end{aligned} \quad (13)$$

This is obtained from the infinitely reducible κ -symmetry [17], with $\delta_{\kappa} \theta_i^{\alpha} = \kappa_{\dot{\alpha}i} \Pi_{--}^{\dot{\alpha}\alpha}$, $\delta_{\kappa} \bar{\theta}^{\dot{\alpha}i} = \Pi_{--}^{\dot{\alpha}\alpha} \bar{\kappa}_{\alpha}^i$, by using the relation $\Pi_{--}^{\dot{\alpha}\alpha} \sim \bar{\lambda}^{\dot{\alpha}} \lambda^{\alpha}$ which provides the general solution of the equations of motion for the bosonic spinor field λ (which is an auxiliary field in the action (12)). The κ -symmetry reduces the number of degrees of freedom to the same $8+(16/2)$ of the twistor string (10).

The simplest way to check the equivalence [14] of (12) to the first, supertwistor part of the Siegel twistor string action (10), is to use Leibniz's rule to move the derivative to act on λ and to take into account that the Penrose-Ferber incidence relations (8) provide the general solution of the constraint (11).

The fact that the twistor string is tensionless [8] can be understood by observing the conformal invariance of the action (10) or (12), which implies the absence of any dimensionful parameters in it. The Berkovits open twistor string model (ii) is equivalent to the tensionless superstring moving in the direct sum of two copies of the $D = 4$, $N = 4$ superspace [14].

4 On the tensionful parent of the twistor string

The tensionless nature of the twistor string was first noticed by Siegel [8], who also posed the problem of searching for a possible tensionful parent. Its existence can also be understood [14] as a consequence of the results in [15, 16] according to which the mass spectrum of the intrinsically tensionless or null string is continuous, while the also tensionless quantum twistor string [1, 7, 8] is assumed to describe the Yang-Mills theory amplitudes and, hence, must have massless fields in the quantum state spectrum. In fact, since in the conformal algebra the dilatation operator does not commute with the square of the momentum operator, a continuous mass spectrum or a zero-masses one are the only alternatives for a conformally invariant theory. The quantization [18] of the tensionless superstring, which leads to massless fields in the spectrum, requires the (explicit or implicit) use of stringy oscillators, which are the suitable variables for a tensionful string. In contrast, those of the null-string are rather the spacetime coordinates and momenta. As a result, the quantization of the twistor string should correspond to the quantization of the tensionless limit of a tensionful superstring, rather than that of the intrinsically tensionless, or null, superstring.

In ref. [8], Siegel discussed the possible tensionful parents of the twistor superstring in a purely bosonic context, proposing a tensionful QCD string [19] as its bosonic part. The inclusion of fermions brings in new questions. In particular, as far as we assume that the tensionless limit has to be a smooth one, the number of degrees of freedom should not change in this limit and, thus, the number of gauge symmetries should be the same, including the number of fermionic κ -symmetries already mentioned.

A detailed discussion of these questions can be found in [14]. In short, if we were interested just in the $N = 1, 2$ counterparts of the tensionless twistor string, their tensionful counterparts would be the $D = 4$, $N = 1, 2$ Green-Schwarz superstrings, as can be seen in the framework of the spinor moving frame or Lorentz harmonics formulation [20]. In the more interesting $N = 4$ case, the tensionful parent action requires an extension of the bosonic sector of superspace so that, to obtain the twistor string, the tensionless limit has to be accompanied by dimensional reduction. One could conjecture that the tensionful parent of the twistor string is given by the $D = 10$ Green-Schwarz superstring. To describe such a relation in a simple way one would have to use the spinor moving frame or the Lorentz harmonics formulation of the $D = 10$ Green-Schwarz superstring [20, 21].

5 Concluding remarks

We would like to mention that, at the present level of understanding, the Green-Schwarz superstring does not appear as the only possible candidate for a tensionful parent of the twistor string. As discussed in [14], one can also consider supersymmetric string models in enlarged tensorial superspaces [22], which have found applications in higher spin theories [23].

This is not the only possible link between twistor string and higher spin theories. According to [18], the quantization of a tensionless limit of the Green-Schwarz superstring should result in a higher spin theory. On the other hand, as a result of our identification of the twistor string with the zero-tension superstring, the twistor string description of the Yang-Mills amplitudes [1, 7] should also be related to the quantization of a tensionless superstring (a tensionless limit of a superstring). It would be interesting to understand the interrelations and differences between these two quantizations of tensionless superstring(s) in some detail.

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